FH-Kiel
University of Applied Sciences

e-mail: andreas.thiemer@fh-kiel.de

The Collapse of Easter Island

Summary:

The most visible evidence of Easter Island's past glory consists of enormous statues - called "moai" - carved from volcanic stone. However, Easter Island's population suffered from 1400 to 1500 A.D. a sharp decline after a thousand years of peace and prosperity. One simple explanation is that the islanders degraded their resource base to the point that it could no longer support the population. But why did this degradation lead to population overshooting and decline on Easter Island and not on the other Polynesian islands? Brander&Taylor (1998) present a model of renewable resource use, that explains this puzzle by known differences in resource growth rates. Their general equilibrium model contains three central components: Malthusian population dynamics, open-access renewable resources and a Ricardian production structure. Also we consider the model extensions of Dalton&Coats (2000) who emphasize the importance of different property-rights institutions.

1. Important definitions

a_{LH}:	unit labour requirement in the resource sector
b:	birth rate
d:	death rate
F:	fertility function
G_S :	natural growth of S
H:	harvest
H_D:	domestic demand of resource good
H_P:	harvest supplied by producers
K:	carrying capacity (maximum possible size of resource stock)
L:	labour
L_H:	labour used in resource harvesting
M:	manufactured goods (for example monumental architecture, "moai")
M_D:	domestic demand for manufactured goods
p:	price of resource good
r:	intrinsic regeneration rate of resource
S:	resource stock ("forest and soil")
w:	wage

2. The Brander&Taylor model

The change in resource stock over time is:

$$\frac{d}{d\,time}S = G_S - H$$

The **biological growth** of the natural resources follows a logistic functional form :

$$G_S = r \cdot S \cdot \left(1 - \frac{S}{K}\right)$$

The **harvesting production function** is of the Schaefer-typ:

$$H_p = \alpha \cdot S \cdot L_H$$

with

$$\alpha > 0$$

$$\Rightarrow$$

$$a_{LH} = \frac{L_H}{H_p} = \frac{1}{\alpha \cdot S}$$

Good M is produced with constant returns to scale using only labour. By choice of units, on of labour produces one unit of good M. Since good M is treated as a **numeraire** whose pric normalized to one, the **wage** w must equal 1 if good M is produced.

Because of **open access** there is no explicit rental cost for using S. The price of the resourc good must equal its unit cost of production:

$$p = w \cdot a_{LH} = \frac{w}{\alpha \cdot S}$$

The representative consumer has the Cobb-Douglas **utility function**:

$$u = h^{\beta} \cdot m^{1 - \beta}$$

with

$$0 < \beta < 1$$

where h and m are individual consumption of the resource good and the manufactured good. consumer is endowed with one unit of labour. Thus at a point in time the **budget constraint**

$$p \cdot h + m \le w$$

Maximizing utility yields:

$$h = \frac{w \cdot \beta}{p}$$

and

$$m = w \cdot (1 - \beta)$$

Total domestic demand is:

$$H_D = h \cdot L = \frac{w \cdot \beta \cdot L}{p}$$

$$M_D = w \cdot (1 - \beta) \cdot L$$

The **full employment condition** is:

$$L_H + L_M = L$$

$$\Rightarrow$$

$$H_p \cdot a_{LH} + M = L$$

Because supply price equals demand price the **temporary equilibrium resource harvest** b

$$H = \alpha \cdot \beta \cdot L \cdot S$$

Equilibrium output of manufactured goods is:

$M = (1 - \beta) \cdot L$

This implies that manufactures will allways be produced, hence $w = 1$.

Substituting the equilibrium harvest into the change of stock equation yields the following differential equation for the **renewable resource dynamics**:

$$\frac{d}{dtime} S = r \cdot S \cdot \left(1 - \frac{S}{K}\right) - \alpha \cdot \beta \cdot L \cdot S$$

The base rate of population increase is (b-d)<0. But consumption of resource good increase fertility (decreases mortality), and therefore increases the rate of population growth.

$$\frac{d}{dtime} L = L \cdot (b - d + F)$$

with

$$F = \phi \cdot \frac{H}{L}$$

Thus we obtain a further differential equation characterizing **Malthusian population dynam**

$$\frac{d}{dtime} L = L \cdot (b - d + \phi \cdot \alpha \cdot \beta \cdot S)$$

Now the whole model boils down to a two-equation system of differential equations, which a variation of a **Lotka/Volterra predator-prey model**, where human population is the "predator" and the resource stock is the "prey".

3. Parameters

We calibrate the model with the parameter values used by Brander&Taylor, which are cons the knowledge about the past of Easter Island. You may change these parameters later on.

Resource dynamics:

$r := 0.04$

$K := 12000$

$\beta := 0.4$

$\alpha := 0.00001$

Population dynamics:

$b := 0.2$

$d := 0.3$

$\phi := 4$

Time horizon:

$T_{max} := 500$

*(**Hint:** If you change parameters, you have to choose "Compute All" to get correct solutions from Mathcad's symbolic processor!).*

4. Steady state analysis

This system reaches a steady state if

$$\frac{d}{dtime} S = \frac{d}{dtime} L = 0$$

The model exhibits three solutions. The both **corner solutions** (L = 0, S = 0) and (L = 0, S = easily seen by inspection. To compute the **interior solution** we first solve for

$$\frac{\frac{d}{d\,time}L}{L} = 0$$

where the solution S_E is the steady state of the resource:

$$S_E := \left(b - d + \phi\cdot\alpha\cdot\beta\cdot S\right) = 0 \text{ solve}, S \rightarrow 6250.$$

Now we solve

$$\frac{\frac{d}{d\,time}S}{S} = 0$$

This yields a linear function of L in S:

$$\Gamma(S) := r\cdot S\cdot\left(1 - \frac{S}{K}\right) - \alpha\cdot\beta\cdot L\cdot S = 0 \left|\begin{array}{l}\text{solve}, L\\ \text{simplify}\\ \text{float}, 3\end{array}\right. \rightarrow 1.00\cdot10^{4} - .833\cdot S$$

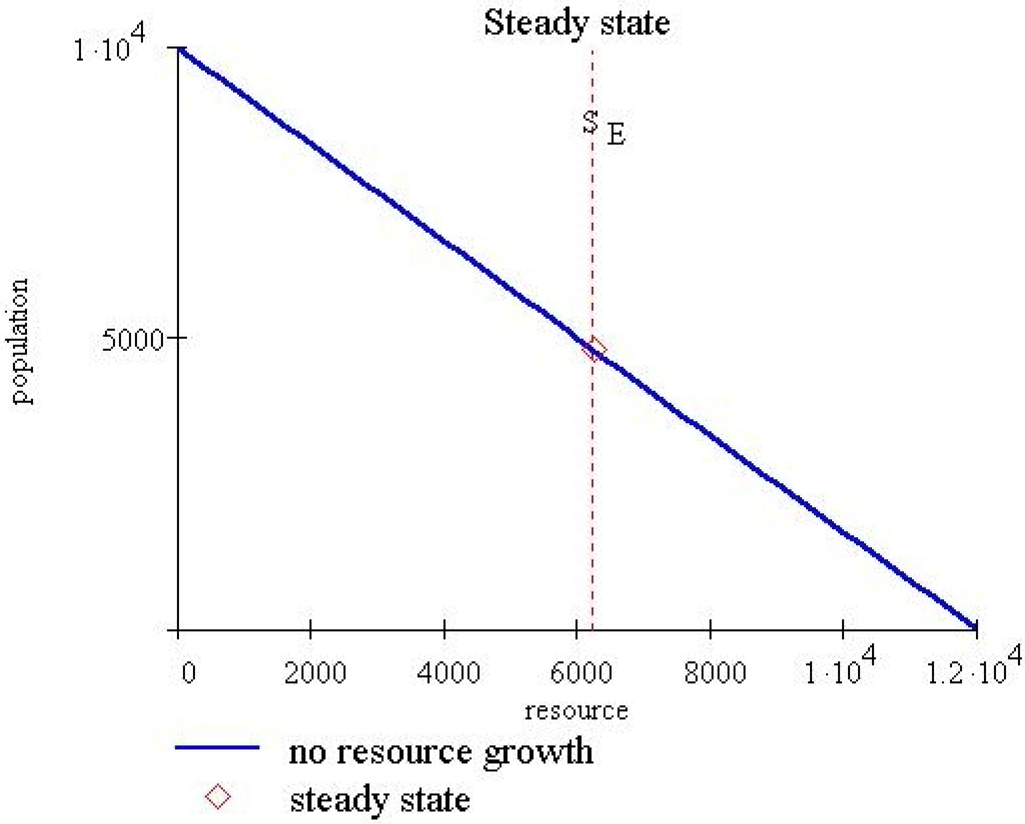
Letting

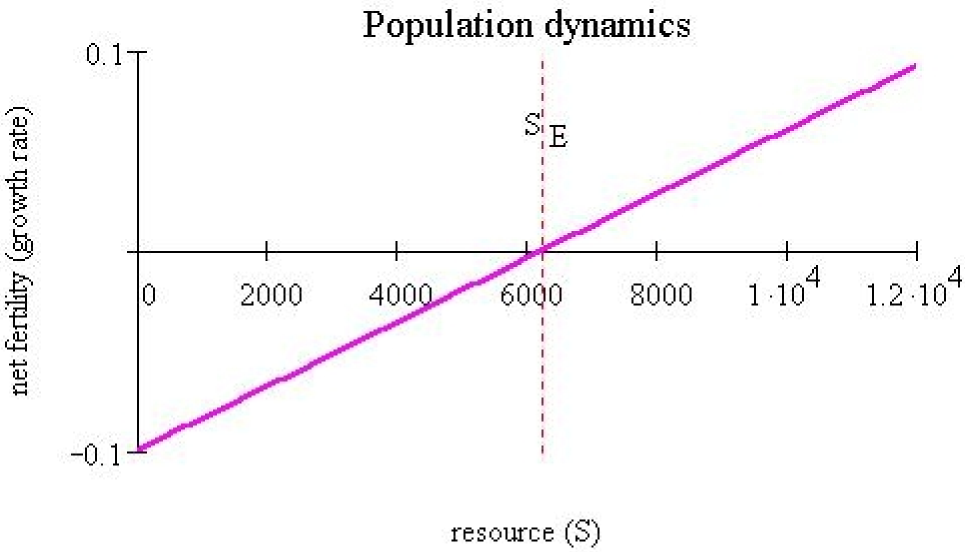
$$L_E := \Gamma(S_E)$$

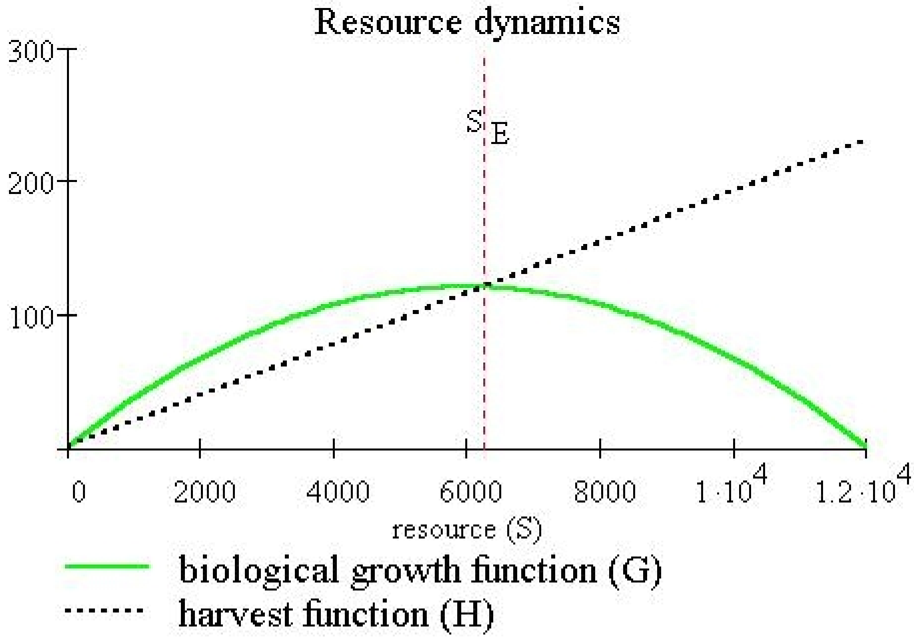
we obtain the (interior) steady state of the population:

$$L_E = 4.794\cdot10^{3}$$

For a graphical representation of this steady state solution see the figures below.







5. Important propositions

Here are some important propositions quoted from Brander&Taylor (for proofs see their ar
"The steady state resource stock

- rises if the mortality rate rises, the birth rate falls, or fertility responsiveness falls;
- falls if there is technological progress in harvesting;
- is unaffected by changes in the intrinsinc resource regeneration rate r, or carrying capacity, K."

"The steady state population level

- rises equiproportionately with an increase in the intrinsic rate of resource growth r;
- falls when harvesting and technology improves if $S < K/2$ and rises if $S > K/2$;
- falls when the taste of the resource good rises if $S < K/2$ and rises if $S > K/2$;
- rises if the carrying capacity of the environment rises."

"When an interior steady state exists, the local behaviour of the system is as follows:

- Steady state 1 ($L = 0$, $S = 0$) is an unstable saddlepoint allowing an approach along the $S = 0$ axis.
- Steady state 2 ($L = 0$, $S = K$) is an unstable saddlepoint allowing an approach along the $L = 0$ axis.
- Steady state 3 ($L > 0$, $S > 0$) is a stable steady state and a 'spiral node' with cyclical convergence if $r \cdot \frac{(d - b)}{K \cdot \phi \cdot \alpha \cdot \beta} + 4 \cdot ((d - b) - K \cdot \phi \cdot \alpha \cdot \beta) < 0$.
- Steady state 3 is a stable state and an 'improper node' allowing monotonic convergence if the inequality from the equation above runs in the other direction."

"When an interior steady state exists, the global behaviour of the system is as follows:

- If $L > 0$ and $S = 0$, the system approaches steady state 1 with $L = 0$ and $S = 0$.
- If $L = 0$ and $S > 0$, the system approaches steady state 2 with $S = K$ and $L = 0$.
- If $S > 0$ and $L > 0$, then the system converges to the interior solution in steady state 3."

Check the validity of the propositions by changing the parameters from above by simulation

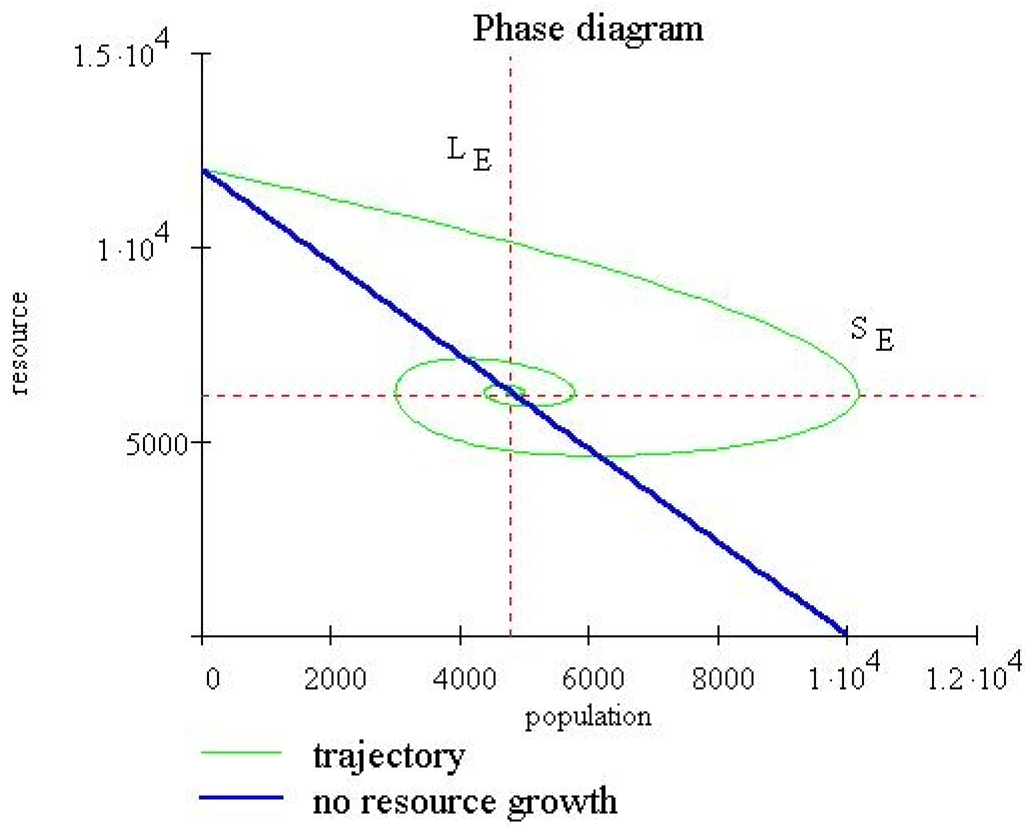
6. Simulation runs

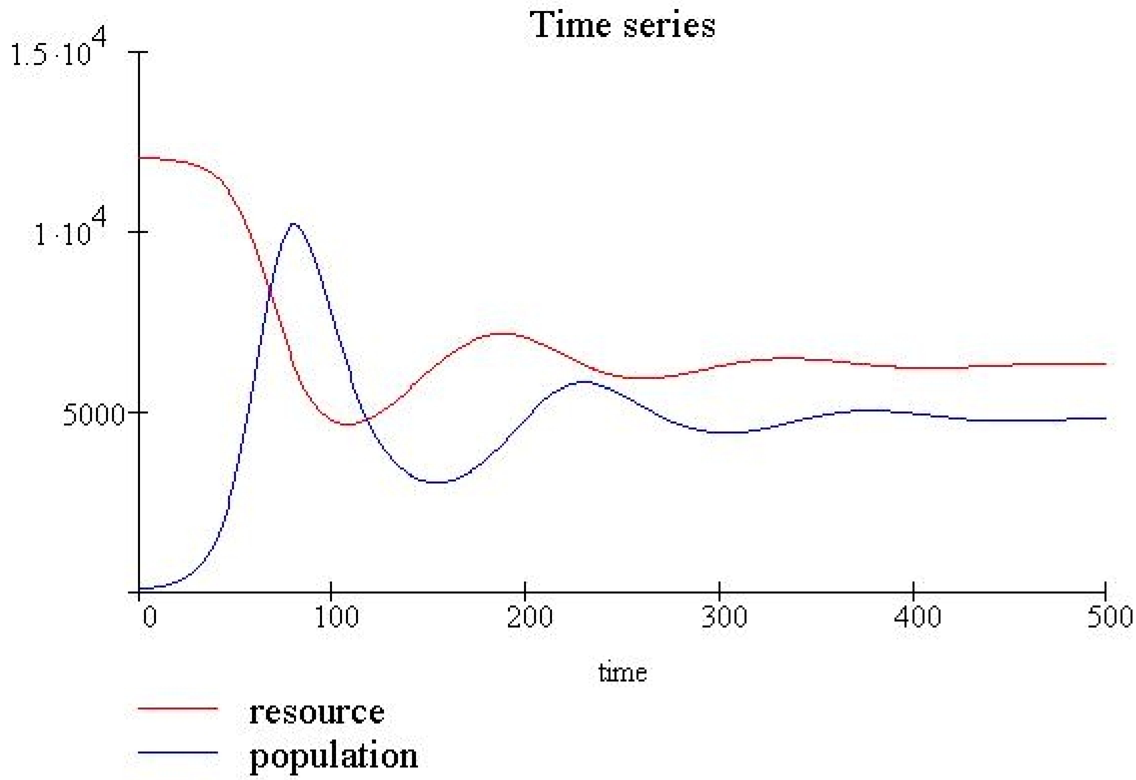
Given the initial values of the resource stock and the labour force, the evolution of these va through time is simulated now.

Initial values:

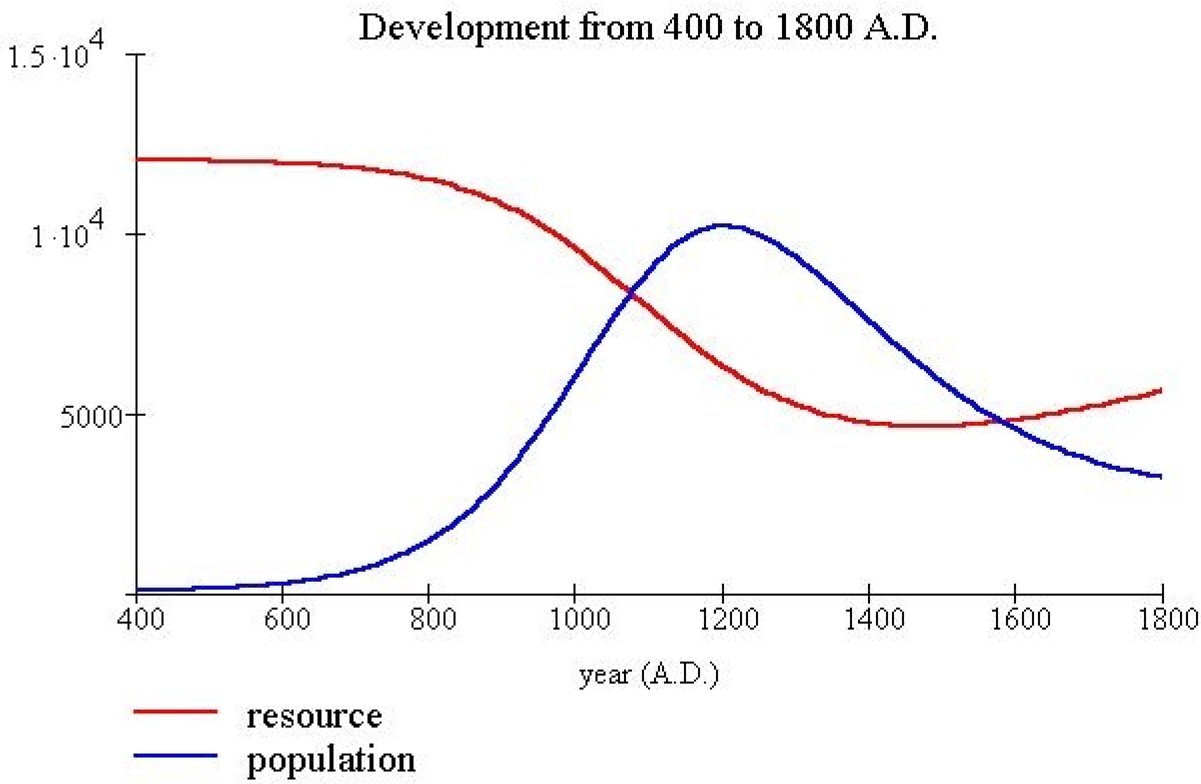
$S_0 := K$

$L_0 := 40$





To interpret the historical development let $T_{max} = 140$. Every time period represents a 10-
intervall, starting from 400 A.D. and ending in 1800 A.D. :



As Brander&Taylor pointed out, the monotonic behaviour of the state of the population on t
Polynesian islands is consistent with their higher intrinsic growth rates of the resource. No
growing trees in Polynesia (coconut palm and Fiji fan palm) can grow on Easter Island. For
based on these palms change the intrinsic growth rate r to 0.35.

Dalton&Coats (2000) modify this model to examine **the impact of different property-righ** by introducing a parameter δ, which "reflects the extent to which a society's institutions enc to alter their behavior in reaction to estimated future prices of the resource." With (dS/dtim surrogate predictor for the future value of the resource good, Dalton&Coats assume that the labour use in harvesting the resource follows:

$$L_H = \beta \cdot L \cdot \left(1 + \delta \cdot \frac{\frac{dS}{dtime}}{S}\right)$$

Under this assumption the resource and population dynamics change to

$$\frac{d}{dtime}S = \frac{r \cdot S \cdot \left(1 - \frac{S}{K}\right) - \alpha \cdot \beta \cdot L \cdot S}{1 + \delta \cdot \alpha \cdot \beta \cdot L}$$

[Note: There is an omission in formula (14) of Dalton&Coats (2000), which is corrected here!]

$$\frac{d}{dtime}L = L \cdot \left[b - d + \phi \cdot \alpha \cdot \beta \cdot S + \delta \cdot \phi \cdot \alpha \cdot \beta \cdot S \cdot \frac{\left[r \cdot \left(1 - \frac{S}{K}\right) - \alpha \cdot \beta \cdot L\right]}{1 + \delta \cdot \alpha \cdot \beta \cdot L}\right]$$

The function $\Gamma(S)$, which is the isocline for $\frac{d}{dtime}S = 0$ remains unchanged, but for $\frac{d}{dtime}$ the new isocline:

$$\Lambda(S) := \phi \cdot \frac{r}{K \cdot (b - d)} \cdot S^2 + \frac{-\phi - \delta \cdot \phi \cdot r}{\delta \cdot (b - d)} \cdot S + \frac{-1}{\delta \cdot \alpha \cdot \beta}$$

Dalton&Coats distinguish three cases of poperty-rights institutions:

1. $\delta = 0$ (**consumption rights**): This is the Brander&Taylor case. There exists a communa of the resource good under the guidance of a chief or ruling council, which permitted lit for individual exploitation of the resource. Such a tribal society focus only on current c and ignores the future availability of the resource.

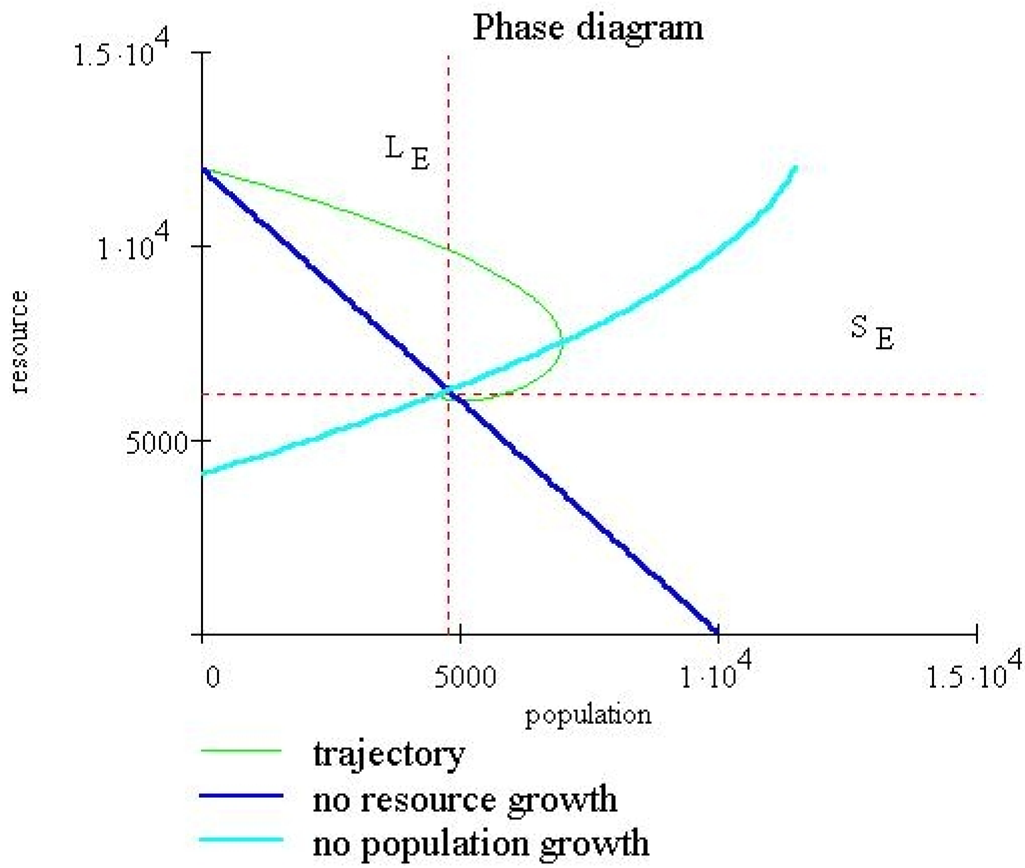
2. $\delta < 0$ (**common-access**): Every individual has unlimited access to the resource. But the the society face the prisoners' dilemma: "While cooperation and self-regulation benefit community as a whole by preserving resource stocks, personal gains accrue by cheating agreed-upon harvest quotas while personal losses accumulate when others cheat."

3. $\delta > 0$ (**private property**): Here the private owners are encouraged to economize on pres harvesting of the resource in response to higher predicted future prices.

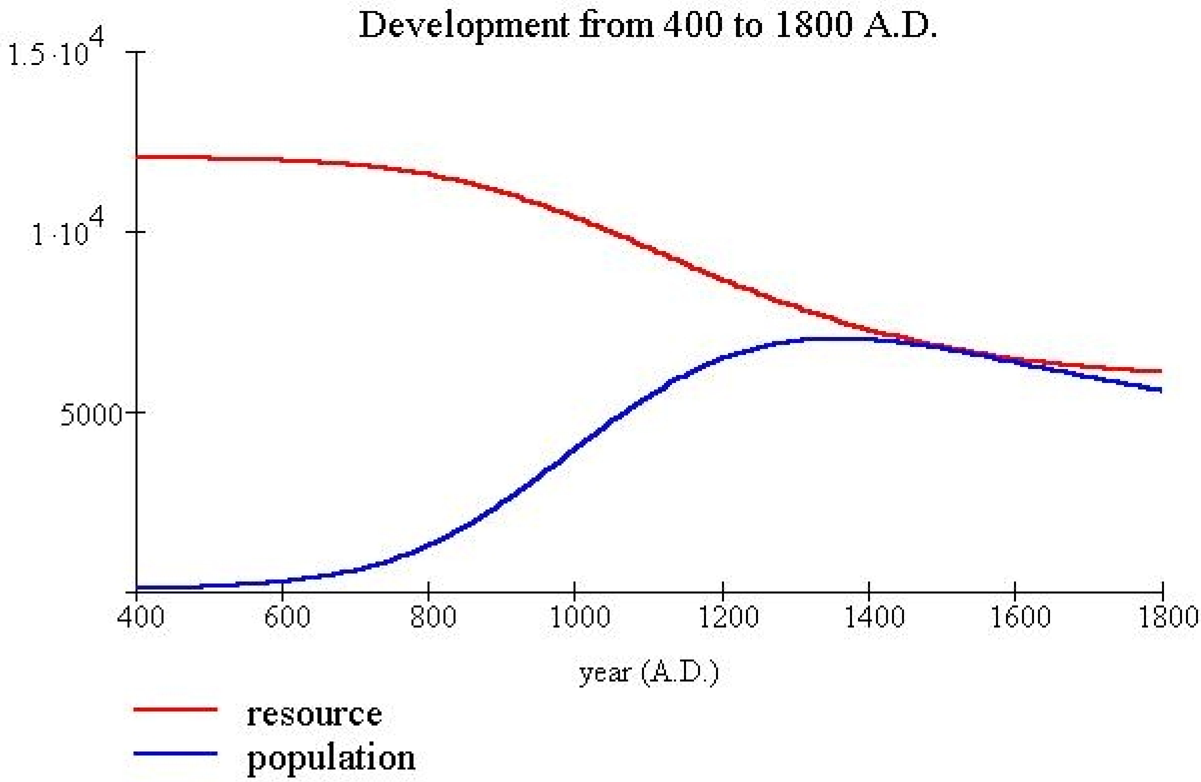
The main conclusions of the Dalton&Coats specification are:

- "All societies can eventually achieve similar steady states."
- "Together with exclusive personal ownership, markets contribute to a stable path to the
Common access ownership destabilizes the adjustment process, creating greater cyclica
economic welfare."

Now repeat the simulation with different values of the "institutional parameter" δ [for exam
Dalton&Coats (2000) used -5 and 20], to verify their statements.



Institutional
parameter:
$\delta \equiv 20$



Literature:

- Brander, J.A./Taylor, M.S.: The Simple Economics of Easter Island: A Ricardo-Malthus Model of Renewable Res Use. In: The American Economic Review, vol. 88 (1998), pp. 119 - 138.
- Dalton, R.T./Coats, R.M.: Could Institutional Reform Have Saved Easter Island? In: Journal of Evolutionary Econo vol. 10 (2000), 489 - 505.